

## Drag Reduction in Dilute Polymer Solutions

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### Synopsis

The problem of drag reduction in a helically coiled tube is examined experimentally. The general problems involved in the correlation of drag reduction data are analyzed critically, and some important recommendations are made based on the findings of this work. A phenomenological approach is suggested for correlating the drag reduction data. An examination of the straight-tube data from the literature and the coiled-tube data obtained in this work shows that for a variety of aqueous polymer solutions the data could be uniquely correlated by the following equation:

$$\beta = 0.2 + \frac{0.8}{1 + (De')^{0.8}}$$

where  $\beta$  = reduced friction factor and  $De'$  = modified Deborah number.

### INTRODUCTION

The problem of drag reduction under turbulent flow conditions by the addition of small quantities of high molecular weight polymers has been extensively studied in the literature. The majority of the work has been done in straight tubes, and there is little information in the literature on drag reduction under conditions when the fluids are flowing in flow situations other than straight tubes. Many tentative explanations have been offered to explain the phenomenon of drag reduction but none of them is entirely convincing. Many phenomenological and molecular approaches have been used to correlate the drag reduction data but a critical analysis of these approaches has not been attempted before.

In view of the factors mentioned above, we have undertaken an experimental study of drag reduction in one such interesting situation, namely, in helically coiled tubes. We have critically analyzed the general problem of correlation of drag reduction data and we have proposed a phenomenological approach for correlating such data. We have then shown that such a correlation enables us to correlate the data for different dilute aqueous polymer solutions flowing in a variety of flow situations.

### BACKGROUND

The study of the flow of dilute polymer solutions in helically coiled tubes is of a great practical significance, since coils are extensively used in in-

dustrial practice for the purpose of cooling and heating liquids. The coils have an advantage over straight pipes in that they occupy lower floor area and provide higher heat transfer coefficients. The knowledge of the reduction in pressure drop under such conditions is of an obvious practical significance.

There are reasons to believe that the behavior of polymer solutions in coiled tubes may be substantially different from that in the straight tubes. The curvature of the tube produces a variation of centrifugal force along the radial direction in a coil. This in turn produces a secondary flow in the cross section of the pipe as a result of which the fluid near the top and the bottom moves inward and the fluid in the middle moves outward. This motion, superimposed on the linear motion of the fluid along the tube, results in a unique flow pattern called the "double helical streamlined flow." This secondary circulation may bring in some interesting observations on drag reduction.

Further, there are some experimental findings in the literature which tend to suggest that the dilute polymer solutions behave in a strange manner when they flow in situations other than a straight pipe. Thus, Pisolkar<sup>1</sup> has shown that when these fluids flow through valves and fittings there is a "drag enhancement" rather than a drag reduction. Barnes and Walters<sup>2,3</sup> and Walters et al.<sup>4</sup> studied the problem of flow of such fluids in curved pipes with gentle curvatures. They showed that there was drag reduction under laminar flow conditions and that there was an enhancement in the drag reduction under transitional conditions, but under properly turbulent conditions the curvature of the tube had an adverse effect on the drag reduction. The theoretical interpretation of drag reduction proposed by Lockett<sup>5</sup> appeared to support the latter observations. However, an analysis of the previous work appeared to show some apparent anomalies. Hence we thought that it will be desirable to reexamine the problem experimentally.

### Mechanism and Correlation of Drag Reduction in Straight Tubes

In spite of the extensive literature which has appeared in the past, there does not seem to be an entirely convincing explanation of the mechanism of drag reduction. Among several different postulated mechanisms there appears to be a general agreement that the phenomenon is a result of the interaction of turbulence and viscoelasticity. Different mechanisms postulate different modes of such interaction. In all the attempts to correlate the drag reduction data quantitatively, a single dimensionless number, sometimes called "Deborah number," is used. This is defined as

$$De = \frac{\text{characteristic fluid time}}{\text{characteristic flow time}} \quad (1)$$

The significance attached to the characteristic flow time depends upon the postulated mechanism. However, the net form of the parameter is remarkably similar in all the cases.

Astarita<sup>6</sup> postulated that the effect of viscoelasticity was to make the energy-dissipating eddies conservative, a situation which arises when the frequencies of the energy-dissipating eddies are higher than the inverse characteristic fluid time. The characteristic flow time is hence taken as the reciprocal of the frequency of energy-dissipating eddies,  $\omega$ , which is calculated as<sup>7</sup>

$$\omega = (U/D)\text{Re}^{0.75} \quad (2)$$

where  $U$  = linear velocity,  $D$  = tube diameter, and  $\text{Re}$  = Reynolds number. Seyer and Metzner<sup>8</sup> assume that the viscoelastic properties of the liquids result in the reduction of the intensity of turbulence due to a general resistance to "stretching." The characteristic flow time is then calculated as being proportional to the stretch rate, and its reciprocal is proportional to  $\omega$ , which is defined in eq. (2).

Gordon<sup>9</sup> assumes that drag reduction occurs because of the strong resistance of the polymer solutions to dispersion or breakup and that the consequent reduction in the viscous dissipation is due to the reduced bursting of the fluid elements ejected from the wall. The characteristic flow time is hence interpreted as the reciprocal of the frequency of such ejections which is quantitatively of the same form as in eq. (2). Meek and Baer<sup>10</sup> use the concept of a periodic sublayer at the bounding surface of the turbulent shear flow and interpret the characteristic flow time as the reciprocal of the mean frequency of penetrations of turbulent fluctuations. Once again the reciprocal flow time is exactly the same as defined in eq. (2).

The analysis of Denn and Porteous<sup>11</sup> makes use of a dimensionless group based on the ratio of the friction velocity  $u^*$  to the shear wave velocity  $\sqrt{(\nu/T)}$ , where  $\nu$  = kinematic viscosity and  $T$  = characteristic fluid time. The resulting dimensionless group for correlation is

$$\text{De}_1 = u^* \sqrt{T/\nu} \quad (3)$$

Hershey and Zakin<sup>12</sup> have taken the characteristic flow time as the reciprocal of the shear rate at the wall, and the Deborah number has been formulated as

$$\text{De}_2 = (u^{*2}/\nu)T \quad (4)$$

The definitions of the Deborah numbers in eqs. (3) and (4) are equivalent. Further, it is evident that if the characteristic fluid time  $T$  is separated from eqs. (3) and (4), then the term  $u^{*2}/\nu$  could be identified with  $U/D \cdot \text{Re}^{0.75}$  defined in eq. (2) (provided the friction factor-Reynolds number relationship for drag-reducing polymer solutions is approximated by a Blasius-type relation, viz.,  $f \propto \text{Re}^{-0.25}$ ).

The agreement shown above over the quantitative definition of the characteristic flow time unambiguously defines the effect of the velocity as well as of the diameter. However, in order to correlate the effect of polymer characteristics (such as molecular weight, concentration, etc.),

a proper definition of the characteristic fluid time is necessary. There does not seem to be a general agreement over the way in which the characteristic fluid time should be defined. Thus, Astarita et al.<sup>7</sup> considered it to be an unspecified constant. Elata et al.<sup>13</sup> assumed it to be a constant given by the Rouse theory.<sup>14</sup> Hershey and Zakin<sup>12</sup> assumed it to be the Zimm relaxation time, and Sayer and Metzner<sup>8</sup> assumed it to be a shear-dependent function given by the "convected Maxwell model."<sup>15</sup> These definitions introduce many uncertainties. For instance, the dependence of the characteristic fluid time on concentration is predicted differently by using different approaches. Thus, using the Zimm theory, one gets the natural time as a weak function of the concentration, whereas Frish and Simha<sup>16</sup> show that it should increase as the square of concentration. The results of Astarita et al.<sup>7</sup> substantiate this for dilute solutions of polyacrylamide (ET597). However, Seyer and Metzner<sup>8</sup> have shown that the variable relaxation time for the same polymer solution decreases with increasing concentrations at higher concentrations. The disagreement as regards the concentration dependence of the natural time has been concisely summarized by Middleman.<sup>17</sup> It should be emphasized that the success of the correlation for different concentrations of a given polymer system will largely depend on the exact way in which the concentration dependence of the characteristic fluid time is taken into account.

#### **Proposed Method of Correlation of Drag Reduction Data in a Helically Coiled Tube**

Earlier discussion and analysis have clearly indicated the importance of the proper definition of a characteristic flow time and a characteristic fluid time. It is proposed that the periods of the high-frequency eddies are probably most representative of the characteristic flow time. Since the turbulence originates near the wall in the region adjacent to the laminar sublayer, the steep velocity gradient in this region must contribute to the production of eddies. On this basis we propose to take the reciprocal of the average shear rate at the wall as the characteristic flow time.

The magnitude of this could be easily evaluated from the friction factor-Reynolds number expression given by Shrinivasan et al.<sup>18</sup>:

$$f = \frac{0.084 (D/D_c)^{0.1}}{\text{Re}^{0.2}} \quad (5)$$

where  $f$  = friction factor,  $D$  = tube diameter, and  $D_c$  = diameter of the helix. The reciprocal of the characteristic flow time will hence be given by

$$\omega = u^{*2}/\nu = (D/D_c)^{0.1}(U/D)\text{Re}^{0.8} \quad (6)$$

Following Astarita et al., we take the characteristic fluid time  $T$  as an unspecified constant. The Deborah number may then be defined as

$$\text{De} = \omega T = (D/D_c)^{0.1}(U/D)\text{Re}^{0.8}T. \quad (7)$$

The definition of  $\omega$  given in eq. (6) may not be generally valid under drag reduction conditions. However, to avoid the necessity of forming an implicit relationship, we have approximated the definition of  $\omega$  in accordance with the general approach used by previous workers.<sup>7,8,11</sup>

From dimensional considerations it follows that the friction factor will be a function of both the Reynolds and the Deborah numbers. Astarita et al.<sup>7</sup> suggested that any explicit dependence of  $f$  on  $Re$  should be removed by using a reduced friction factor  $f/f_0$ , where  $f_0$  is the friction factor under purely viscous conditions. We then have

$$f/f_0 = \beta(De). \quad (8)$$

It has been observed that  $\beta(De)$  is a monotonically decreasing function of  $De$ , with an asymptotic value of 0.2 to 0.4 at very high values of  $De$ . Examining the individual factors in the definition of Deborah number in eq. (7), it is clear that, among other factors, an increase in the curvature ratio  $D/D_c$  will help in enhancing drag reduction. This is contradictory to the observations made by Walters et al.<sup>4</sup> We have, however, analyzed their data later and shown that the conclusion reached in their work was not generally valid. In fact, it is to be expected that an increase in any factor which results in an increase in the Deborah number should generally help in enhancing the drag reduction.

In order to eliminate the difficulty due to the uncertainty in the value of  $T$ , we will consider the point of the  $\beta(De)$  curve where  $\beta$  has a constant value of  $k$ . It follows that

$$k = \beta(\omega_k T) = \beta(C) \quad (9)$$

where  $C$  is a constant not depending on the particular solutions considered. We then have an alternative functional form given by

$$\beta = \beta(\omega C/\omega_k) = \beta'(\omega/\omega_k). \quad (10)$$

By using eq. (10), it is possible to correlate the data for different solutions in different diameter tubes by a single curve in the form of  $\beta'(\omega/\omega_k)$ . The technique used here was originally proposed by Astarita et al.<sup>7</sup>; but they defined a fixed value of  $k$  as 0.5. However, one could choose any value of  $k$ , provided it is sufficiently smaller than 1. In fact, an analysis of the data of Astarita et al. with drag ratio of 0.6 and 0.7 showed that the same form of the  $\beta'(\omega/\omega_k)$  curve could be obtained as the one obtained by taking the drag ratio as 0.5. It is fairly difficult to obtain reduction ratios as high as 0.5 for all polymer concentrations, and hence we chose a value of 0.6 for  $k$ .

## EXPERIMENTAL

A single helically coiled tube made of copper was used for all the studies. The coil tube had an internal diameter of 1.25 cm and a curvature diameter of 66.5 cm; the pitch of the coil was 3.8 cm. The total length of the coil was 1255 cm, and the number of turns were 6. The mean inside diameter

calculated before and after coiling by determining the internal volume of water indicated that the effect of coiling was not significant.<sup>18</sup> The pressure drops were measured with manometers. The manometric test solutions were carbon tetrachloride and mercury in the lower and higher region of flow rates, respectively. The pressure drop readings were severely scrutinized for reproducibility to avoid the danger of degradation of polymer solutions. Readings were taken when the flow rates were increased as well as decreased.

The solutions used were aqueous solutions of polyacrylamide, PAA (AP30) and PAA (ET597), supplied by Dow Chemicals. Only the range of concentrations usually considered relevant for drag reduction (viz., 50–500 ppm) was studied. Aqueous solutions of poly(ethylene oxide) (Polyox WSR 301, supplied by Union Carbide) were also used. Unfortunately, the reproducibility of the results for poly(ethylene oxide) solutions was rather poor due to degradation, and hence the data have not been considered further.

The shear stress–shear rate data for all the solutions were taken on a Weissenberg rheogoniometer (Model R-18). The data were taken both before and after the run to ensure further the absence of any degradation. The agreement in flow curves in both the cases was always satisfactory.

## RESULTS AND DISCUSSION

### Analysis of Data on Helically Coiled Tube

The accuracy of the measurement of pressure drops in the coiled tube was first examined by taking data for water and checking the values of friction factors as a function of Reynolds number with the correlations reported by Shrinivasan et al.<sup>18</sup> An excellent agreement was found between the two values.

The rheograms of the eight dilute solutions of PAA taken in this work indicated that the solutions showed slight shear thinning behavior in the range of concentrations between 200 and 500 ppm. For a valid basis of comparison, the friction factor–Reynolds number data for such solutions must lie on the same line as that for water in the region which could be approximated by a straight pipe. This region generally exists in the case of helically coiled tubes for Dean numbers ( $Re\sqrt{D/D_c}$ ) less than approximately 30.<sup>18</sup> Hence the generalized Reynolds number defined by Dodge and Metzner<sup>19</sup> was used. The power-law constants of various solutions are reported in Table I and this clearly shows that it would be unwise to choose a constant viscosity, particularly at higher concentrations.

Figures 1 and 2 show the friction factor–Reynolds number data obtained in the case of polyacrylamide (AP-30 and ET-597) solutions. The straight line in the region where  $Re < 200$  corresponds to the region where the behavior of the coil is approximated by a straight pipe. The straight line drawn is based on the relation  $f = 16/Re$ , and the data obtained in

TABLE I  
Rheological Properties of the Polymer Solutions Used in this Work

Solution	Consistency index $K$ , g/(cm sec <sup>2-n</sup> )	Power-law index $n$
ET 597		
50 ppm	0.0108	1.00
100 ppm	0.0133	1.00
200 ppm	0.0344	0.91
500 ppm	0.1800	0.76
AP 30		
50 ppm	0.0111	0.99
100 ppm	0.0195	0.93
200 ppm	0.0440	0.86
500 ppm	0.2800	0.75

this work show a good agreement. Unfortunately, only 200- and 500-ppm solutions were used because the lower concentrations did not enable us to obtain an accurate measurement of pressure drop in the low Reynolds number range.

The data in the laminar region in the coil are now examined. It was found that the generalized Reynolds number used enabled a plot of the

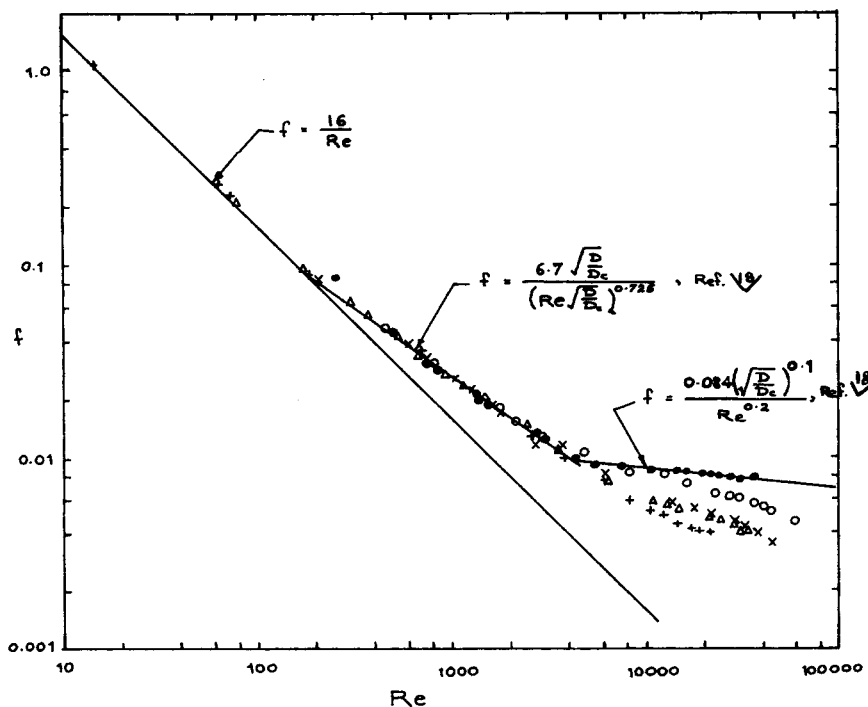


Fig. 1. Friction factor vs. Reynolds number data for PAA (ET 597) solutions in helical coil: (●) water; (○) 50 ppm; (×) 100 ppm; (△) 200 ppm; (+) 500 ppm.

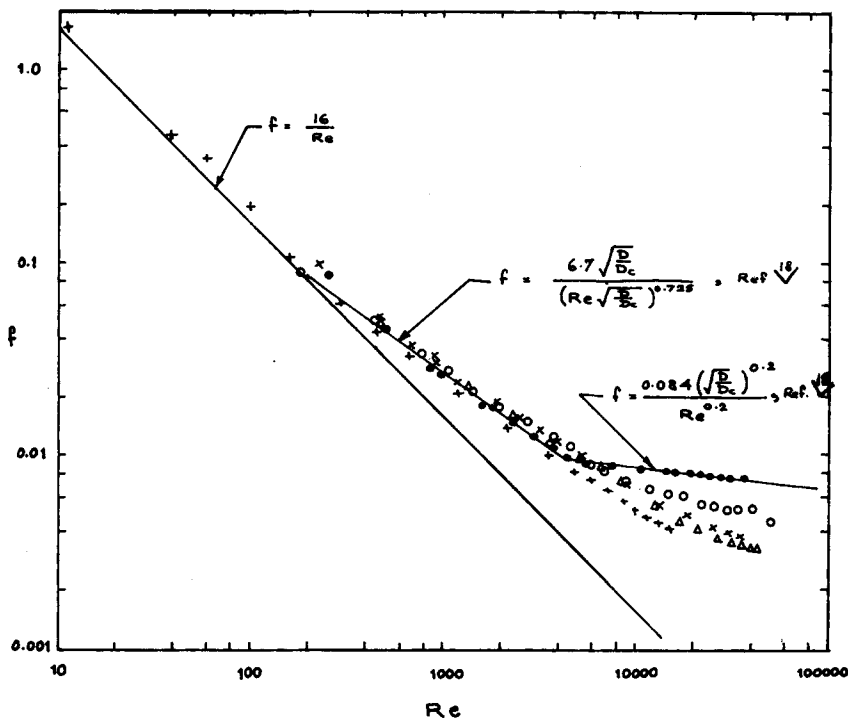


Fig. 2. Friction factor vs. Reynolds number data for PAA (AP 30) solutions in helical coil: (●) water; (○) 50 ppm; (×) 100 ppm; (△) 200 ppm; (+) 500 ppm.

friction factor-Reynolds number data on the same curve as that for water or other Newtonian solutions. This indicated that there was no drag reduction under laminar flow conditions in a coil. Recent work<sup>20</sup> on the flow of poly(methyl methacrylate) solutions in bends under laminar flow conditions substantiates our results.

These observations are in contradiction to the observations made by Barnes and Walters<sup>2,3</sup> and Walters et al.<sup>4</sup> They showed that there was a drag reduction in the curved pipe even when the conditions were laminar. They regarded the conditions of swirling flow in the coil as a "pseudoturbulence" and postulated that it is not necessary to have properly turbulent conditions to achieve drag reduction, but that merely the presence of the secondary flow could cause reduction in drag. It is possible that the only effect of the enhanced laminar circulation could be to increase the shear rates at the wall which will effectively reduce the apparent viscosity. However, the generalized Reynolds number used in this work suitably accounts for the effect of shear thinning viscosity and shows that the friction factor-Reynolds number curves could be plotted on a single curve. The Reynolds number used in the work of Walters et al.<sup>4</sup> incorporates a viscosity term which is taken to be the same as zero shear viscosity. Any possibility of shear thinning in the range of high shear rates in helical coils may tend to show a different behavior.



It is not possible to compare the data obtained for the dilute polymer solutions in the laminar region with the theoretical predictions which are available in the literature. The theoretical analysis done by Jones<sup>21</sup> and Thomas and Walters<sup>22</sup> excludes the second-order effects of curvature which will strongly contribute in the region we have studied. Further, the constitutive equation used by these authors was equivalent to a third-order Coleman-Noll fluid. It has been shown elsewhere<sup>23</sup> that this model describes the fluid behavior under very restricted flow conditions and will certainly not be applicable in the range of shear rates encountered in this work.

Figures 1 and 2 show the data for different polymer solutions under turbulent conditions, and the uniform decrease in friction factors with increasing concentration of polymers is evident. It will be useful to compare the observations made in the present work on drag reduction in the turbulent flow regime in a coiled tube with those available in the literature. Walters et al.<sup>4</sup> are the only workers who have attempted to study the problem of drag reduction in curved tubes. They showed that there was an increase in drag as the curvature was increased and that any advantage obtained in terms of drag reduction in straight pipes would be lost if the pipes were sufficiently curved. Since we have worked only with a single coil of fixed internal diameter and fixed curvature ratio, it is not possible to verify these conclusions. It is, however, possible that different results could be obtained, particularly with tightly wound helices.

### Correlation of Drag Reduction Data in Straight and Coiled Tubes

Having examined the flow behavior of dilute polymer solutions in coiled tubes, we now consider the problem of correlation of the data on drag reduction in both straight and coiled tubes.

The important problem of the characteristic fluid time is first examined. The friction factor data obtained in this work were replotted as  $\beta$  versus  $\omega$ , and the values of  $\omega_{0.6}$  were calculated from these curves. The quantity  $1/\omega_{0.6}$  is proportional to the characteristic fluid time. Some useful conclusions may be drawn by examining the variation of the characteristic fluid time with respect to the concentration of the polymer solution. Figure 3 shows a plot of  $1/\omega_{0.6}$  versus concentration. It is readily seen that the dependence of  $1/\omega_{0.6}$  on concentration tends to decrease as the concentration is increased. In fact, at higher concentrations the characteristic fluid time tends to be practically independent of concentration. This is not surprising since the characteristic time is a measure of the ratio of the elasticity and the viscosity of the material, and consequently the exact mode of the variation of the characteristic time will largely depend on the way elasticity and viscosity individually change as the concentration is increased. At lower concentration, the elasticity increases quite substantially without significantly affecting the viscosity level. However, above a certain concentration both elasticity and viscosity may vary in a quite similar manner.

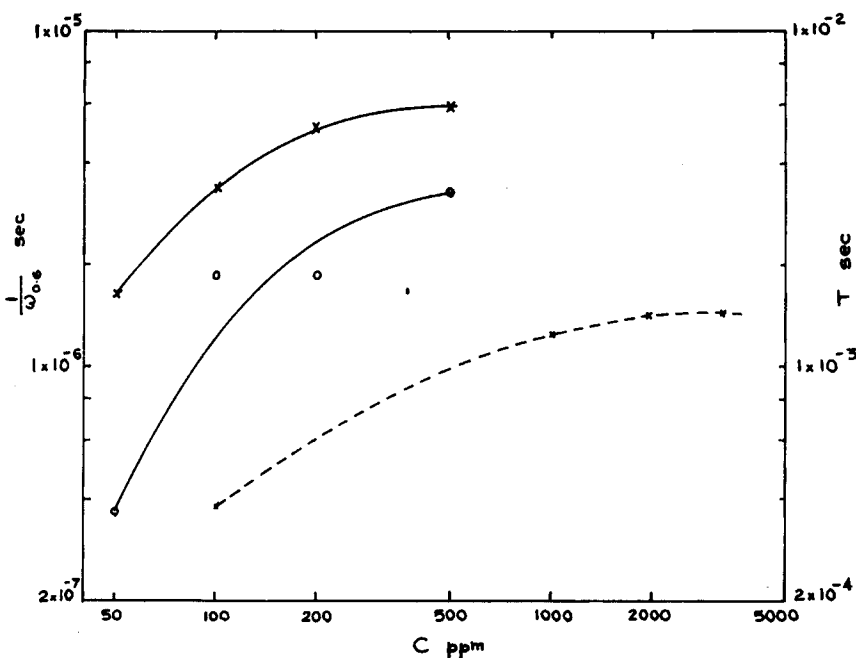


Fig. 3. Dependence of fluid relaxation time on concentration: (X) PAA (ET 597); (O) PAA (AP 30); (---) PAA (ET 597), from rheological data of references 8 and 24 (estimated at a shear rate of  $10^4 \text{ sec}^{-1}$ ).

Due to the experimental difficulties involved, there are very few data available in the literature on the values of fluid relaxation times for the systems used in this work. Oliver<sup>24</sup> and Seyer and Metzner<sup>8</sup> have reported some data on the fluid relaxation times of polyacrylamide (ET-597) solutions in the range of concentrations from 100 to 5000 ppm. Figure 3 shows the values of the fluid relaxation times calculated at a shear rate of  $10^4 \text{ sec}^{-1}$  and plotted versus the concentration of polymer solution. There is a remarkable similarity in the concentration dependence of  $1/\omega_{0.6}$  and the fluid relaxation time. Further, at a given concentration, the quantity  $1/\omega_{0.6}$  for polyacrylamide (ET-597) appears to be approximately 2.5 times greater than that for polyacrylamide (AP30) solutions. Since  $1/\omega_{0.6}$  is directly proportional to the fluid relaxation time, it would appear that the fluid relaxation times for the two solutions should change in the same proportion. A comparison of the fluid relaxation times reported by Bruce and Schwarz<sup>25</sup> for polyacrylamide (AP-30) solutions and those of Oliver<sup>24</sup> for polyacrylamide (ET-597) solutions shows that this is indeed the case. No such comparison has been made in the past. Our observations do indicate the possibility of using a turbulent flow rheometer for estimating the fluid relaxation times of dilute polymer solutions.

It is worthwhile commenting briefly on the choice of a characteristic fluid time for the correlation of drag reduction data. In the absence of experi-

mental relaxation time data, many workers have used the estimates based on molecular theories. However, all the molecular theories predict relaxation times which increase monotonically with the concentration of the polymer solution. This is contrary to the trend shown by available experimental data on fluid relaxation times. Further, this is contrary to the experimental observation that for fixed flow conditions all polymer solutions show maximum drag reduction effect at a particular concentration, and any further increase in concentration over this optimum value actually increases the drag. This would indicate that the fluid relaxation time must show a similar optimum value with respect to the concentration. Thus, the use of molecular theory estimates in correlating drag reduction data over an extended range of concentrations may be of doubtful validity.

The use of a variable relaxation time<sup>8</sup> also appears to be somewhat suspect. The method of correlation used by Astarita et al.<sup>7</sup> and Peterson and Beckwith<sup>26</sup> clearly indicates that the diameter effect in drag reduction could be satisfactorily taken into account by correlating the reduced friction factor with a Deborah number containing a constant fluid characteristic time. If a shear-dependent fluid relaxation time was the proper choice, then it would depend on  $U/D$ . This will show a further dependence on the diameter, and the correlation will fail in that the data for different diameter tubes could not then be correlated by using a single curve.

Figures 4 and 5 show the plot of  $\beta$  versus  $De' = \frac{\omega}{\omega_{0.6}}$  for AP-30 and ET-597 solutions over approximately a 150-fold range of  $De'$ . It is interesting to observe that both types of polymer solutions could be correlated by a unique

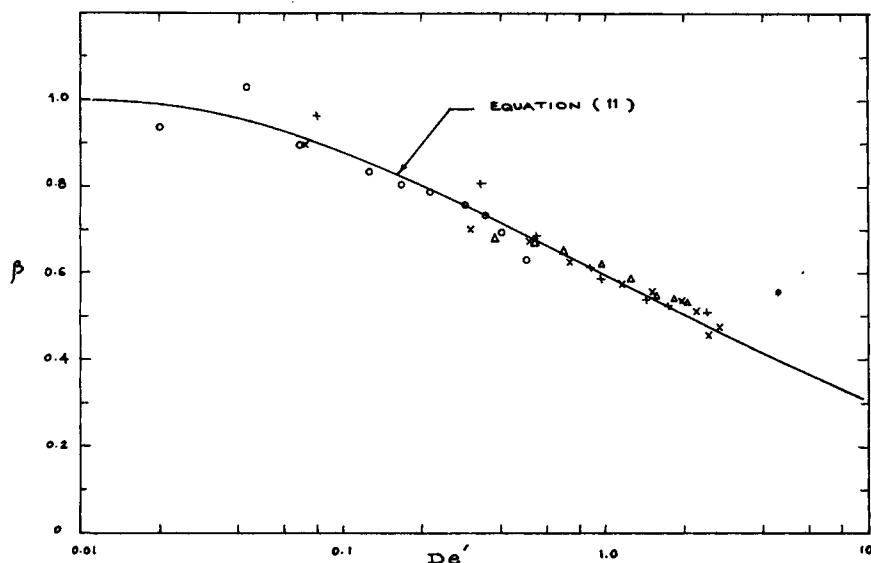


Fig. 4. Reduced friction factor vs. modified Deborah number curve for PAA (ET 597) solutions in helical coil: (O) 50 ppm; (X) 100 ppm; ( $\Delta$ ) 200 ppm; (+) 500 ppm.

curve of  $\beta(De')$ . It would be desirable to compare the drag reduction in a straight pipe and a coil. It is inappropriate to compare the data at the same Reynolds number because of the substantial delay in transition in the case of a coiled tube. We analyzed the straight-tube data reported by Astarita et al.<sup>7</sup> on polyacrylamide (ET-597) solutions, of Whitsit et al.<sup>27</sup> on polyacrylamide (AP-30) solutions, and of Pisolkar<sup>28</sup> on guar gum solutions. These data were recalculated so as to obtain the  $\beta(De')$  curve. Figure 6 shows the comparison of these data. Excellent agreement between different solutions was found. Peterson and Beckwith<sup>26</sup> have already shown that there is good agreement between the curve based on the work of

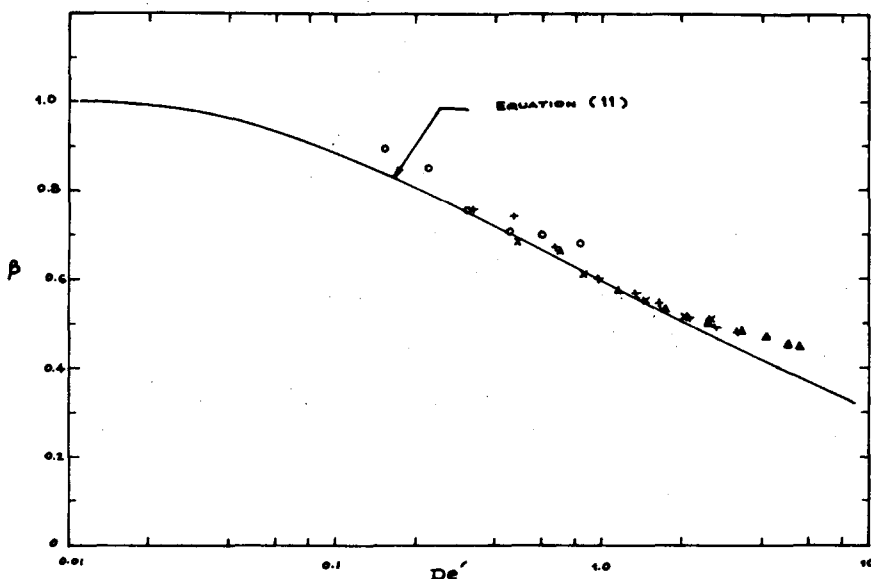


Fig. 5. Reduced friction factor vs. modified Deborah number for PAA (AP 30) solutions in helical coil: (O) 50 ppm; (X) 100 ppm; ( $\Delta$ ) 200 ppm; (+) 500 ppm.

Astarita et al. and their curve based on the data on poly(ethylene oxide) (WSR 301 and WSR 295) solutions (only for polar solvents). This indicates that a unique correlation for different polymer solutions in polar solvents can be obtained.

In Figure 6 we also compare the recalculated curve based on the straight-tube data of Astarita et al. and the curve based on our work on the helically coiled tube. In spite of the difference in the hydrodynamic characteristics in the two cases, there is an excellent agreement between the two curves.

Mashelkar<sup>29</sup> has analyzed the data on drag reduction in external rotational flows (disc, cone, and cylinder) based on a method similar to that used in this work. It is also remarkable to note that the reduced power number-versus-modified Deborah number curves are quite similar to those obtained in this work. A significant conclusion could be drawn that the

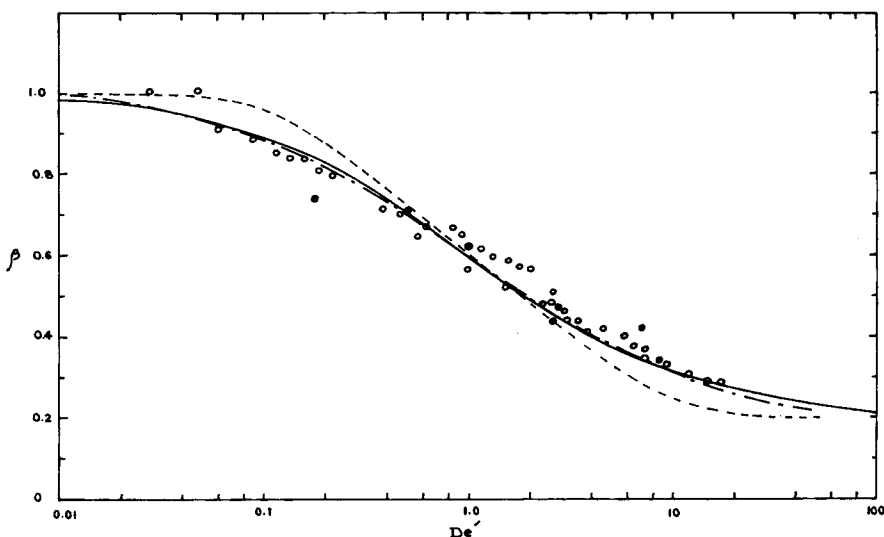


Fig. 6. Comparison of reduced friction factor vs. modified Deborah number relation for different polymer solutions in helical coil and straight pipes: (●) PAA (AP 30) in water (10, 100, and 1000 ppm) in straight pipe, reference 27; (○) guar gum in water (100, 200, 300, and 400 ppm) in straight pipe, reference 28; (—) correlation by Walsh<sup>29</sup>; (---) PAA (ET 597) in water (125, 250, 500, 1000, and 2000 ppm) in straight pipes, reference 7; (—) PAA (AP 30) and PAA (ET 597) in water (50, 100, 200, and 500 ppm) in helical coil, eq. (11).

phenomenon of drag reduction in helically coiled tubes could be predicted based on the observations in a straight tube. In fact, if the results of the present work are further substantiated by extensive experimental work, then it may be possible to obtain unique correlations for the phenomenon of drag reduction for a wide range of internal and external flow situations. An empirical correlation for the dependence of  $1/\omega_k$  with concentration will be sufficient for predicting the drag reduction for different polymer systems in a variety of flow situations.

A correlation which could be valid for predicting the drag reduction data for different polymer solutions in polar solvents in straight tubes and helically coiled tubes was attempted. It was found that the following equation was able to correlate the data quite well:

$$\beta = 0.2 + \frac{0.8}{1 + (De')^{0.8}} \quad (11)$$

Since there are very few theoretical interpretations of predictive values, it may be interesting to compare the results of some of them with the correlation obtained in this work. Walsh<sup>30</sup> has proposed a mechanism for drag reduction in dilute polymer solutions. He proposed that polymer molecules slightly alter the energy balance of the turbulent fluctuations close to the wall and allow viscous dissipation to destroy disturbances which

could have sufficient energy to flow had the polymer molecules not been present. He proposes to correlate the drag reduction defined by

$$L = \frac{T_w - T_s}{T_w - T_p} \quad (12)$$

where  $T_w$  = wall shear stress for the solvent,  $T_s$  = wall shear stress for the solution, and  $T_p$  = extrapolated value of wall shear stress for laminar flow condition. Further, a parameter  $H$  is defined which is the ratio of the contribution of the polymer molecules to the energy balance to the net rate of turbulent energy being transported by turbulent fluctuations from the sub-layer toward the main flow. Walsh found that  $H$  could be defined by

$$H = \frac{SCM[\eta]^2 T_s}{RT} \quad (13)$$

where  $C$  = concentration of the polymer in solution,  $M$  = molecular weight,  $[\eta]$  = intrinsic viscosity, and  $R$  = gas constant. Equation (13) could be rearranged to give

$$H = \left(\frac{T_s}{\mu}\right) \left(\frac{SCM[\eta]^2 \mu}{RT}\right) \quad (14)$$

where  $\mu$  = solution viscosity.

The factor  $T_s/\mu$  in eq. (14) is the shear rate at the wall and is consequently the reciprocal characteristic flow time defined in our work. The second factor can be easily identified as a constant dependent on polymer properties alone and is proportional to the molecular relaxation time. It is further seen that the term  $L$  is approximately equal to  $(1 - \beta)$  at a higher Reynolds number. Thus, the correlation between  $L$  and  $H$  proposed by Walsh is formally the same as that between  $(1 - \beta)$  and the Deborah number in our work. Figure 6 shows the shape of such a correlation (recalculated) obtained by Walsh. There is a reasonably good agreement between his correlation and our correlation. It is evident that since the net correlating form of the dimensionless numbers as well as the shape of the resulting correlation is quite similar, no matter what mechanism of drag reduction has been postulated, it is fairly difficult to support one or the other postulate as being more correct than the other. Alternative methods such as flow visualization or the study of heat or mass transfer in such systems may serve as the proper probes for further understanding of the mechanism of drag reduction.

## CONCLUSIONS

1. The problem of drag reduction is examined experimentally in a helically coiled tube. Contrary to previously published results in the literature, it is shown that there is no drag reduction under laminar flow conditions and that there is drag reduction under turbulent flow conditions.

2. A phenomenological approach is suggested for the correlation of drag reduction data which uses a constant unspecified fluid characteristic time. The following correlation between the reduced friction factor and the modified Deborah number for different aqueous polymer solutions was obtained:

$$\beta = 0.2 + \frac{0.8}{1 + (De')^{0.8}}$$

3. The problem of correlation of drag reduction data is examined critically. It is shown that the different postulated mechanisms give rise to the same form of dimensionless correlation parameters. The problem of the proper definition of the fluid characteristic time is examined, and it is shown that its concentration dependence does not conform to any of the existing molecular theories. The variation of the unspecified characteristic fluid time obtained in this work is shown to be identical with the experimentally determined values of the fluid relaxation time. These observations suggest the possibility of using a turbulent flow rheometer for the evaluation of the fluid relaxation time.

4. By examining the straight-pipe data from the literature, it is shown that the equation derived above, uniquely correlates the data for different aqueous polymer solutions in different flow situations.

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